Optimization Heuristics to Solve Bipartition Graphs

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Abstract. This paper makes an analysis and a comparison between three heuristics, which give approximations to solution in polynomial time for Optimization problems like the graph bipartition problem. The heuristics uses are: Extreme Optimization (EO) searching in solution space the variable with poorest value for a random update to optimize the problem, Extreme Optimization Distributed (EOD) that is a generalized version of EO searching in a neighborhood and Extreme Optimization Distributed with Small World (EODsw) that searches the variables with poorest value in a neighborhood applying Small World. For the analysis we used Geometric, Random and Small World graphs. The obtained results show us that EOD and EODsw are very similar in their behavior and are better than EO, but EODsw result more competitive than EOD for some classes of problems. Key Words: Optimization, correlation, Small World.

1 Introduction

The Bak-Sneppen model [1] is based over species, it associates one value, which is knows as "fitness" taking values between 0 and 1; it is called value of adaptability; the species with the smallest value (poorest degree of adaptation), is selected for a random update, with the purpose of improving its adaptability. Change species adaptation values affects at his neighbors. The adaptability of the neighbors could be modify selecting another random value, warrant work only in the space of solutions; after sufficient number of iterations the system reaches a highly correlated state known as criticality self – organized. The Extreme optimization (EO) is based over the Bak-Sneppen model, only that here, the element with the smallest value, is selected for a random update without any explicative improve for it.
The Extreme Optimization Distributed model (EOD) searches the element with the poorest adaptation inside a locality, then, performs a random update. The Bak-Sneppen model takes the element with the poorest adaptation from a global set, nevertheless EOD takes the element from a locality or a neighborhood.
The Small World (SW) it is a kind of EOD because is based in a neighborhood. "You are only ever six degrees of separation away form anybody else on the planet", Social networks model six 'degrees of separation' (1990) [2], explains that anybody are six persons of separation at most. Small Worlds is a generalized version of this model.
This paper is organized as follows. In section 2 we describe the techniques and the implementation of the used algorithms. In section 3 we present the Graph Bipartition Problem. In section 4 we show the result of the computer simulation of heuristics

2.4 Extreme Optimization Distributed with Small World

There are three fundamental aspects in the study of the network, which have the property to be large and sparse, according James Case [3]:

(a) The average k over all vertices v of the number of edges incidents on v.
(b) The average over all connected pairs of different vertices, of the length L of the shortest connecting path.
(c) The frequency C with which three connected vertices are completely connected.

Extreme optimization distributed with Small World (EODsw) works in the same way EOD with neighborhood, the structural properties of the graphs measures for its characteristically length of path L, average over all vertexes. Moreover grouping coefficient C defined like: suppose that a vertices v has k_v neighbors; then can exist at most $k_v(k_v-1)/2$ edges between them (this occur when all neighbors of v are connected with all) C denotes the fraction of edges allowed, existence. L is the typical separation measure between two vertex in the graph (one property global), while C is the measure typically grouping from neighborhood (one local property). The regular networks have a big C but small L while the random networks have a big L but small C. There are some networks between this class that have small L and big C. One network with this property improves performance to search because it can find better local optima because high grouping C, but at the same time have a relation with all the system, to compare between this minimum, due to small L. The small world networks used here have a lot of vertex with scarce connection, but not the sufficient to disconnect the graph. Specific is required where guarantees that the random graph are connected. Exist scarce connection, facilities the implementation in distributed ambient, because the number of message interchange are small O(k).

3 The Graph Bipartition Problem

The problem of graph bipartition is a problem that we can to solve with EO. The Graph Bipartition Problem (GBP) is: there is a graph of N vertices, where N is even, have to separate the vertices in two sets of cardinality N/2, such that the number of edges that connect both sets "cut size" m, is small.

To approximate this problem for a solution using EO, we associate each vertex v_i one variable x_i can take two values 0, 1, indicated belong to one of two sets. The adaptation function of each variable x_i is defined as follows:

$$f_i = \frac{g_i}{g_i + b_i} \tag{3}$$

Were g_i, is the number of "good" edges that connect 'i' with others vertices inside the same set, and b_i is the number of "bad" edges that connect 'i' with vertices through partition. For nodes not connected, we associate one $f = 1$. In order to optimize the problem, we can minimize the cut size of the graph.

4 Results of Computer Simulation

We used different type of graphs to prove the heuristics, like this: random graphs, geometric graphs and small world graphs, varying in each one the vertexes and probability or ratio with tow vertexes are connected; the implementation of small world graphs can be found in Ref [8].

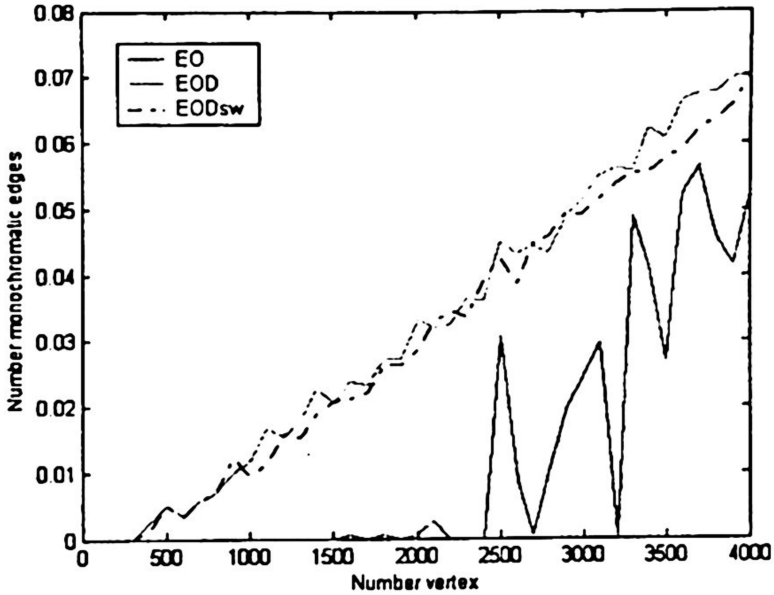


Fig. 1. Geometrics graphs (Number monochromatic edges & number vertex) with 4000 vertex and probability of 0.1

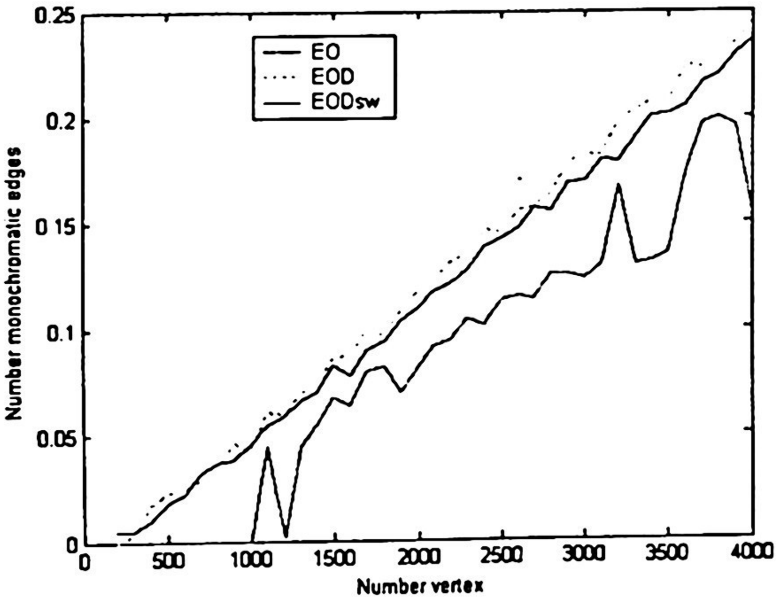


Fig. 2. Random graphs (Number monochromatic edges & number vertex) with 4000 vertex and probability of 0.001

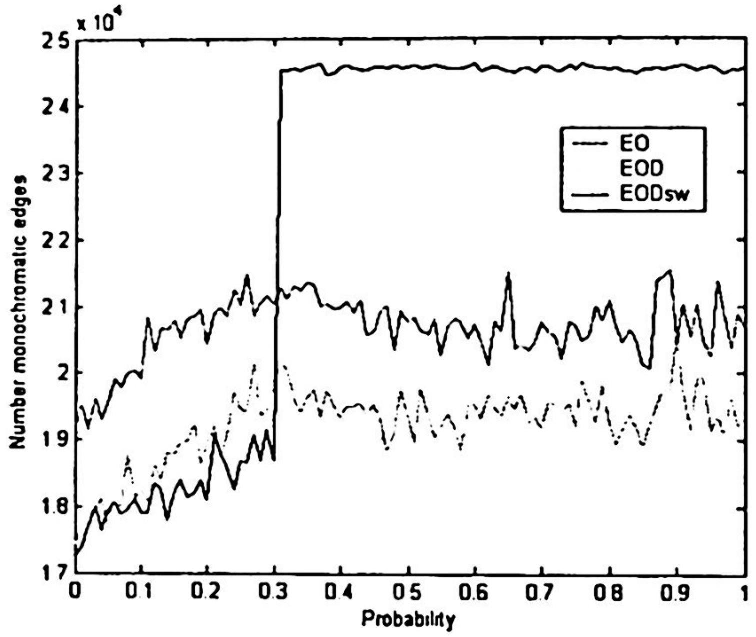


Fig. 3. Small World graph (Number monochromatic edges & Probability) with 2000 vertex

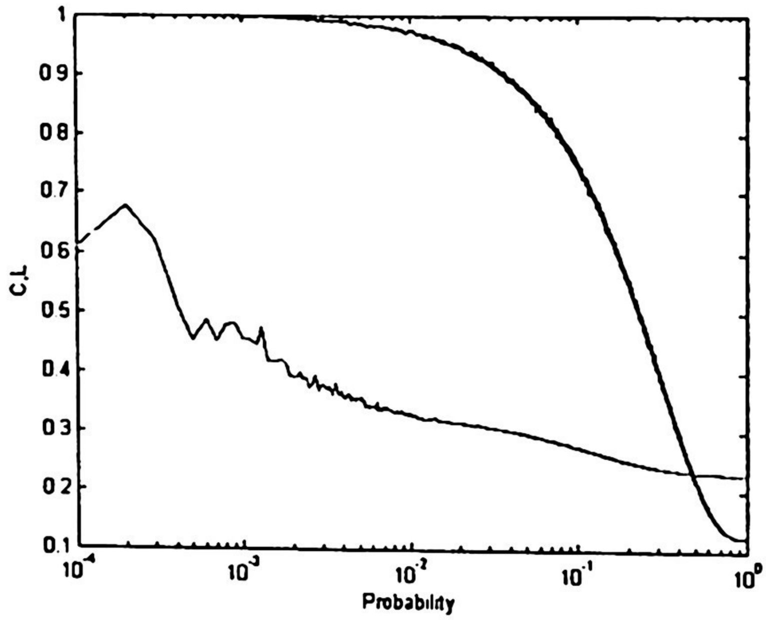


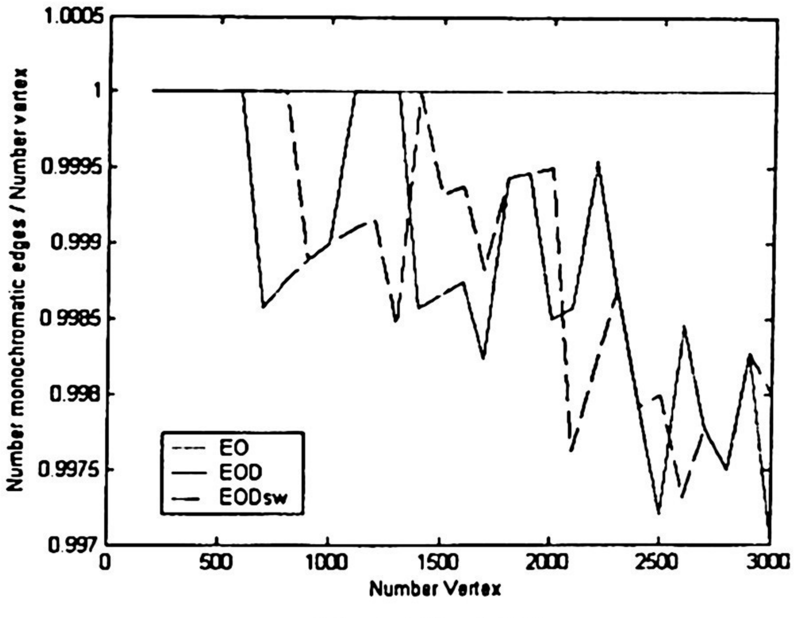
Fig. 4. Small World graph (CL & Probability) with 2000 vertex

In the graphics, it can be observed that EOD and EODsw are very similar, even in fig. 3 EODsw is better that EOD because it will work over SW graph. We can conclude for this part that EOD is better that EO and EODsw is better that EOD over one SW graph; then in normal conditions EOD and EODsw are similar. Now analysis the Bipartition Graph problem:

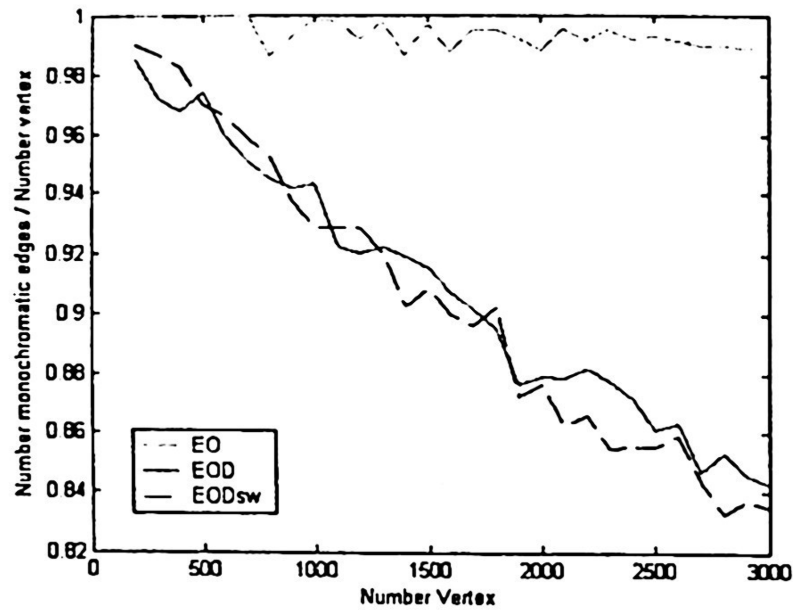
Table 1. Comparisons between EO, EOD and EODsw. We can see that the values between two distributions are similar, sometimes EODsw is better that EOD sometimes not, but if we follow the behavior of each algorithm with 500, 1000, 1500... we can distinguish how are varying before 3000 vertex

Geometrics Graphs with 3000 vertex						
Prob/ Radio	Distribution 0.001			Distribution 0.01		
	EO	EOD	EODsw	EO	EOD	EODsw
0.5	1	0.9975	0.9985	0.985	0.85	0.83
0.6	1	0.997	0.998	0.985	0.845	0.835
0.7	1	0.9984	0.998	0.985	0.84	0.825
0.8	1	0.9977	0.999	0.986	0.849	0.82
0.9	1	0.999	0.9975	0.985	0.839	0.83
1.0	1	0.9984	0.999	0.985	0.85	0.83
1.1	1	0.9985	0.9975	0.986	0.845	0.83
1.2	1	0.9975	0.9975	0.986	0.845	0.83
1.3	1	0.9965	0.9974	0.986	0.8455	0.825
1.4	0.999	0.99755	0.999	0.987	0.84	0.83
1.5	0.88	0.999	0.999	0.88	0.83	0.84
1.6	1	0.9992	0.9974	0.985	0.845	0.825

As you can see, for this analysis we use a Geometric graph for the computer simulation varying the distributions of the vertex and the probability or ratio with tow vertexes are connected, next some representatives figures from Table 1.



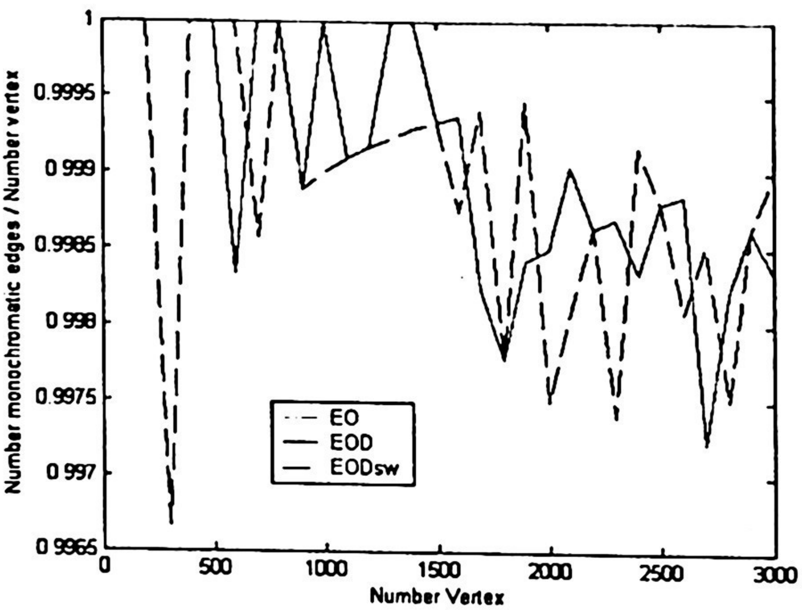
a) Probability 0.001



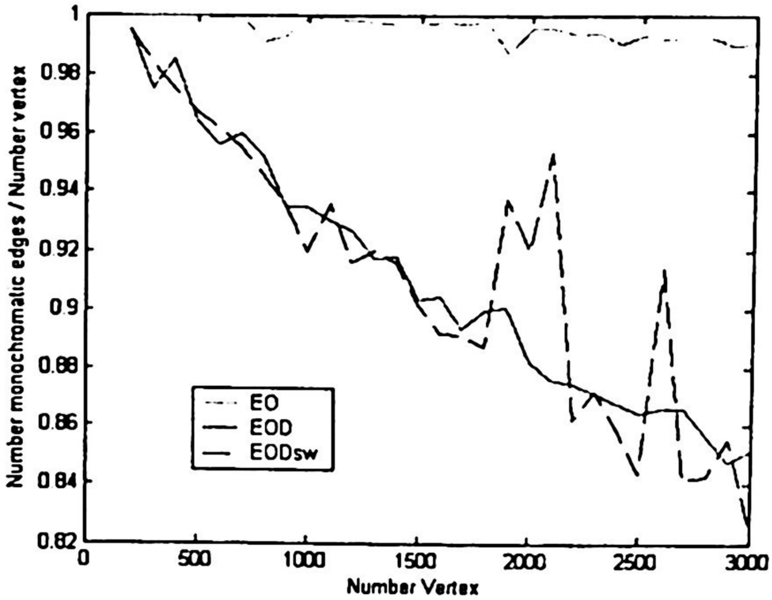
b) Probability 0.01

Fig. 5. Bipartition graphs problem (Number monochromatic edges / Number vertex & Number Vertex) with 3000 vertex and distribution 0.6

In fig. 5 EO don't follow the others, EOD and EODsw varying but they maintain a similar behavior.



a) Probability 0.001

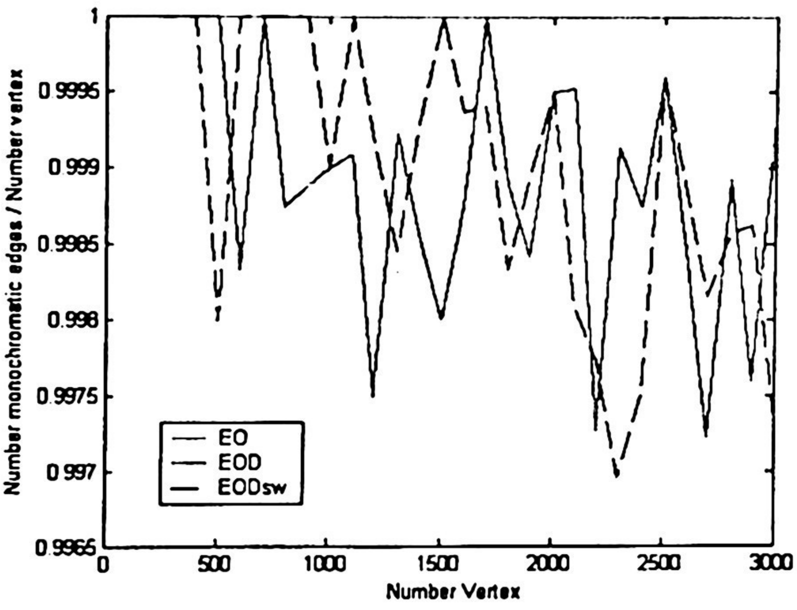


b) Probability 0.01

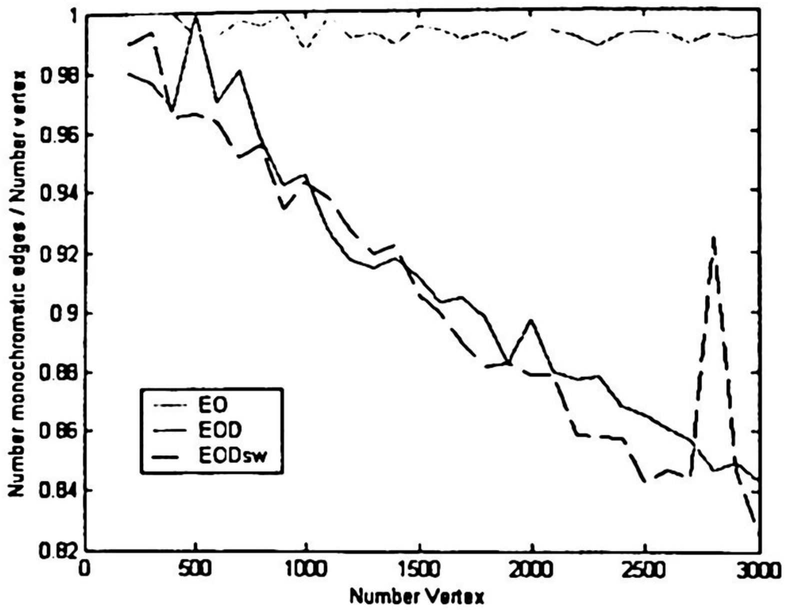
Fig. 6. Bipartition graphs problem (Number monochromatic edges / Number vertex) with 3000 vertex and distribution 1

This figure are very similar to fig. 5, OE remains on top of the graphic while EOD and EODsw maintain their behavior and decreases continuously. Some times EOD is better than EODsw and sometimes EODsw is better.

Next our last figure, his behavior is similar to the other figures; the figures from Table 1 that we don't graphics have a similar behavior like presente here.



a) Probability 0.001



b) Probability 0.01

Fig. 7. Bipartition Graphs Problem (Number monochromatic edges / Number vertex) with 3000 vertex and Distribution 1.6

5 Conclusions

In this work we reviewed three algorithms; first we implemented them and made a comparison to find the best. Then we talk about the Bipartition Graphs Problem and see that sometimes EODsw is better and sometimes EOD is better. Concluding EODsw and EOD are similar to solve the Bipartition Graph Problem, we can apply the specific algorithm for our necessities like in the figure 7) a) 1300 vertex EOD is better that EODsw.

We want to do this comparative to determine that if we apply EODsw we went to obtains better results that EOD and EO, but we didn't, the results obtained tell us that both algorithms in general are better that EO, and not only in bipartition problem. If we have a SW graph, we can take advantage from EODsw, but if we are in bipartition problem we can't. To bipartition problem the used graph is not important, hence is not important the graph's distributions, because when we apply the algorithm and move the vertex to another positions in the graph, we change the topology, the only thing that import us is the connection of the vertex to another vertex, it does not matter if exist a neighborhood in the graph.

In the future works, the same problem it can be solve for three, four, five, et cetera partitions, to try to find if EODsw is better to the other, maybe it can be possible because if we have more partitions we would translate the neighborhood to the partitions.

6 References

1. Stefan Boettcher, Allon Percus "Nature's way of optimizing". Artificial Intelligent 119 (2000) 275-286. ELSEVIER.
2. D.J. Watts, "Small Worlds: The Dynamics of Networks Between Order and Randomness". Princeton University Press, Princeton, NJ, 1999.
3. Case, James. "The Continuing Appeal of Small World Networks". Community Lecture 2001. SIAM News, Vol 34, Number 9.
4. A. Broder, R. Kumar, F. Maghoul, P. Rajagopalan, R. Stata, A. Tomkins, and J. Wiener, "Graph structure of the web". Proceedings of the 9th WWW Conference, Amsterdam, 15-19 May, 2002, 309.
5. S. Bornholdt, H.G. Schuster. "Handbook of graphs and Networks from the Genome to the Internet". Wiley-VCH, Germany, 2003.
6. Kleinberg, Jon. "The Small-World Phenomenon: An Algorithmic Perspective". Department of Computer Science. Cornel University. Ithaca, NY.
7. Kleinberg, Jon. "Small-World Phenomena and the Dynamics of Information". Department of Computer Science. Cornel University. Ithaca, NY, 14853.
8. D. J. Watts, S. H. Strogatz "Collective dynamics of 'small-world' networks". Nature Vol 393, 4/Junio/1998, pp. 440.
9. Buchanan Marrk. "Nexus, Small Worlds and the Groundbreaking Science of Networks". W. Norton & Company. New York. 2002.